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KAON-NUCLEON SCATTERING FROM CHIRAL LAGRANGIANS

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ABSTRACT

The s-wave $K^{\pm}N$ scattering amplitude is computed up to one-loop order corresponding to next-to-next-to-leading order (or N^{2}LO in short) with a heavy-baryon effective chiral Lagrangian. Constraining the low-energy constants by on-shell scattering lengths, we obtain contributions of each chiral order up to N^{2}LO and find that the chiral corrections are "natural" in the sense of viable effective field theories. We have also calculated off-shell s-wave K^-N scattering amplitudes relevant to kaonic atoms and K^- condensation in "nuclear star" matter including the effect of $\Lambda(1405)$. The K^-p amplitude is found to be quite sensitive to the intermediate $\Lambda(1405)$ contribution, while the K^-n amplitude varies smoothly with the C.M. energy. The crossing-even one-loop corrections are found to play an important role in determining the higher-order chiral corrections.

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Ever since Kaplan and Nelson[1] first predicted kaon condensation in dense nuclear matter, there have been numerous investigations on this issue, based both on effective chiral Lagrangians[2, 3, 4, 5, 6] and on phenomenological off-shell meson-nucleon interactions [7, 8, 9]. The results have been quite confusing: While effective chiral Lagrangian treatment leads generally to a robust behavior of the condensation phenomenon, with a rather low critical density of order of three or four times ordinary matter density ρ_0, the phenomenological treatment constrained by empirical kaon-nucleon data seemed to predict otherwise, in some cases leading to no possibility of condensation at all. This raises the question as to whether the chiral Lagrangian that predicts kaon condensation at low enough density is consistent with empirical kaon nucleon and kaon nuclear interactions at low energy. Inconsistency with scattering data would throw in doubt the premise with which the predictions are made, thereby raising questions on the validity of the arguments that go into the exciting scenario of dense "nuclear star" matter and of the formation of pygmy black holes, recently put forward by Brown and Bethe [10].

In this paper, we address, using heavy baryon chiral perturbation theory (HBChPT in short)[11, 12, 13], the question as to whether the chiral Lagrangians used for predicting kaon condensation are able to describe kaon nucleon and kaon nuclear scattering. The advantage of HBChPT is that it allows one to compute higher order chiral corrections systematically, as shown recently in different contexts by several authors [12, 13, 14, 15]. The recent paper by Brown, Lee, Rho and Thorsson[6] (BLRT) addressed this problem to next-to-leading order in chiral counting but this involved only the tree order. In this paper, we will go one step further, that is to N^{2}LO, by including one-loop contributions. Similar one-loop calculations were recently reported by Bernard *et al.* on s-wave pion-nucleon scattering[16]. Focusing on s-wave kaon-nucleon interactions as appropriate to the processes we are interested in, it will be shown that crossing-even one-loop corrections will determine the size of higher-order chiral corrections when the low-energy constants are constrained by on-shell kaon-nucleon scattering lengths. We also calculate the off-shell K^-N scattering amplitude and find that our result is in good agreement with the recent phenomenological fit of Steiner [17].

Following Jenkins and Manohar[12], we first write down the Lagrangian that we shall employ. Let the characteristic momentum/energy scale that we are interested in be Q. The standard chiral counting orders the amplitude as a power series in Q, say, Q^ν, with ν an integer. To leading order, the kaon-nucleon amplitude T^{KN} goes as $\mathcal{O}(Q^1)$, to next order as $\mathcal{O}(Q^2)$ involving no loops and to next to next order (*i.e.*, N^{2}LO) at which one-loop graphs enter as $\mathcal{O}(Q^3)$. In terms of the velocity-dependent octet baryon fields B_v, the octet meson fields $\exp(i\pi_a T_a/f) \equiv \xi$, the velocity-dependent decuplet baryon fields T_v^μ, the velocity four-vector v_μ and the spin operator S_v^μ ($v\cdot S_v = 0$, $S_v^2 = -3/4$), the vector current $V_\mu = [\xi^\dagger, \partial_\mu \xi]/2$ and the axial-vector current $A_\mu = i\{\xi^\dagger, \partial_\mu \xi\}/2$, the Lagrangian density to order Q^3, relevant for the low-energy s-wave scattering, reads

$$\begin{aligned}
\mathcal{L}^{(1)} &= \mathrm{Tr}\ \bar{B}_v(iv\cdot\mathcal{D})B_v + 2D\mathrm{Tr}\ \bar{B}_v S_v^\mu\{A_\mu, B\} + 2F\mathrm{Tr}\ \bar{B}_v S_v^\mu[A_\mu, B] \\
&\quad -\bar{T}_v^\mu(iv\cdot\mathcal{D}-\delta_T)T_{v,\mu} + \mathcal{C}(\bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_v^\mu) + 2\mathcal{H}\bar{T}_v^\mu(S_v\cdot A)T_{v,\mu} \qquad (1)\\
\mathcal{L}^{(2)} &= a_1\mathrm{Tr}\ \bar{B}_v\chi_+B_v + a_2\mathrm{Tr}\ \bar{B}_vB_v\chi_+ + a_3\mathrm{Tr}\ \bar{B}_vB_v\mathrm{Tr}\ \chi_+ \\
&\quad +d_1\mathrm{Tr}\ \bar{B}_vA^2B_v + d_2\mathrm{Tr}\ \bar{B}_v(v\cdot A)^2B_v + d_3\mathrm{Tr}\ \bar{B}_vB_vA^2 + d_4\mathrm{Tr}\ \bar{B}_vB_v(v\cdot A)^2 \\
&\quad +d_5\mathrm{Tr}\ \bar{B}_vB_v\mathrm{Tr}\ A^2 + d_6\mathrm{Tr}\ \bar{B}_vB_v\mathrm{Tr}\ (v\cdot A)^2 + d_7\mathrm{Tr}\ \bar{B}_vA_\mu\ \mathrm{Tr}\ B_vA^\mu \\
&\quad +d_8\mathrm{Tr}\ \bar{B}_v(v\cdot A)\ \mathrm{Tr}\ B_v(v\cdot A), \qquad (2)\\
\mathcal{L}^{(3)} &= c_1\mathrm{Tr}\ \bar{B}_v(iv\cdot\mathcal{D})^3B_v + g_1\mathrm{Tr}\ \bar{B}_vA_\mu(iv\cdot\overleftrightarrow{\mathcal{D}})A^\mu B_v + g_2\mathrm{Tr}\ B_vA_\mu(iv\cdot\overleftrightarrow{\mathcal{D}})A^\mu\bar{B}_v \\
&\quad +g_3\mathrm{Tr}\ \bar{B}_vv\cdot A(iv\cdot\overleftrightarrow{\mathcal{D}})v\cdot AB_v + g_4\mathrm{Tr}\ B_vv\cdot A(iv\cdot\overleftrightarrow{\mathcal{D}})v\cdot A\bar{B}_v \\
&\quad +g_5\left(\mathrm{Tr}\ \bar{B}_vA_\mu\mathrm{Tr}\ (iv\cdot\overrightarrow{\mathcal{D}})A^\mu B_v - \mathrm{Tr}\ \bar{B}_vA_\mu(iv\cdot\overleftarrow{\mathcal{D}})\mathrm{Tr}\ A^\mu B_v\right) \\
&\quad +g_6\left(\mathrm{Tr}\ \bar{B}_vv\cdot A\mathrm{Tr}\ B_v(iv\cdot\overrightarrow{\mathcal{D}})v\cdot A - \mathrm{Tr}\ \bar{B}_vv\cdot A(iv\cdot\overleftarrow{\mathcal{D}})v\cdot A\mathrm{Tr}\ B_vv\cdot A\right) \\
&\quad +g_7\mathrm{Tr}\ \bar{B}_v[v\cdot A,[iD^\mu, A_\mu]]B_v + g_8\mathrm{Tr}\ B_v[v\cdot A,[iD^\mu, A_\mu]]\bar{B}_v \\
&\quad +h_1\mathrm{Tr}\ \bar{B}_v\chi_+(iv\cdot\mathcal{D})B_v + h_2\mathrm{Tr}\ \bar{B}_v(iv\cdot\mathcal{D})B_v\chi_+ + h_3\mathrm{Tr}\ \bar{B}_v(iv\cdot\mathcal{D})B_v\mathrm{Tr}\ \chi_+ \\
&\quad +l_1\mathrm{Tr}\ \bar{B}_v[\chi_-, v\cdot A]B_v + l_2\mathrm{Tr}\ \bar{B}_vB_v[\chi_-, v\cdot A] + l_3[\mathrm{Tr}\ \bar{B}_v\chi_-, \mathrm{Tr}\ B_vv\cdot A], \qquad (3)
\end{aligned}$$

where the covariant derivative $\mathcal{D}_\mu$ for baryon fields is defined by

$$\begin{aligned}
\mathcal{D}_\mu B_v &= \partial_\mu B_v + [V_\mu, B_v], \\
\mathcal{D}_\mu T^\nu_{v,abc} &= \partial_\mu T^\nu_{v,abc} + (V_\mu)_a^d T^\nu_{v,dbc} + (V_\mu)_b^d T^\nu_{v,adc} + (V_\mu)_c^d T^\nu_{v,abd}, \qquad (4)
\end{aligned}$$

δ_T is the $SU(3)$ invariant decuplet-octet mass difference, and

$$\chi_\pm \equiv \xi\mathcal{M}\xi\pm\xi^\dagger\mathcal{M}\xi^\dagger, \qquad (5)$$

with $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ the quark mass matrix that breaks chiral symmetry explicitly. The constants D, F, a_i,..., l_i are determined as described below. The decuplet fields are not written down in $\mathcal{L}^{(2),(3)}$ since they do not figure to N^{2}LO in the s-wave amplitudes we are interested in.

Despite the awesome appearance of the Lagrangian density with its numerous terms contributing, there is a large simplification for the s-wave $K^{\pm}N$ amplitudes. The subleading terms (*i.e.*, terms with $\nu \geq 2$) involving the spin operator S_μ do not contribute to the s-wave $K^{\pm}N$ amplitudes, since they are proportional to $S\cdot q$, $S\cdot q'$, or $S\cdot qS\cdot q'$, all of which vanish. What this means is that there is no contribution to the s-wave meson-nucleon scattering amplitude from one-loop diagrams in which the external meson lines couple to baryon lines through the axial vector currents. Thus we are left with only six topologically distinct one-loop diagrams, Fig.1, (out of thirteen in all) for the s-wave meson-nucleon scattering apart from the usual radiative corrections in external lines. Since we are working to $\mathcal{O}(Q^3)$, only $\mathcal{L}^{(1)}$ enters into the loop calculation. Now $\mathcal{L}^{(2)}$ contributes terms at order $\nu = 2$, receiving no contributions from the loops, as discussed already in [6]. These will be completely given by the KN sigma term and calculable $1/m_B$ corrections. The terms in $\mathcal{L}^{(3)}$, numerous as they are, remove the divergences in the one-loop contributions and supply finite counter terms that are to be determined empirically. As we will mention later, these constants are determined solely by isospin-odd amplitudes, the loop contribution to isospin-even amplitudes being free of divergences.

As discussed in [6], to order Q^2, only the tree diagrams contribute. Here as a first analysis, we extend the argument of [6] by incorporating $\Lambda(1405)$ which plays an important role in the K^-p channel [18]. Let q (q') denote the four-momenta of the incoming (outgoing) mesons and v^μ ($v^2=1$, $v^0>0$) be the velocity four-vector of the nucleons in the limit $m_N\to\infty$. The velocity four-vector of the nucleon assumed to be "heavy" does not change throughout the meson-nucleon scattering as long as the momentum transfer is small enough compared with the nucleon mass [11, 12]. The on-shell s-wave $K^{(\pm)}$ scattering amplitudes at tree level can be readily written down from $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$ with $\hat{m} = (m_u+m_d)/2$:

$$\begin{aligned} f^2\, T_v^{K^{\pm}p} &= \mp\frac{1}{2}v\cdot(q+q') - (a_1+a_2+2a_3)(\hat{m}+m_s) \\ &\quad +\frac{1}{2}(d_1+d_3+2d_5+d_7)q\cdot q' + \frac{1}{2}(d_2+d_4+2d_6+d_8)v\cdot qv\cdot q', \qquad (6) \\ f^2\, T_v^{K^{\pm}n} &= \mp\frac{1}{4}v\cdot(q+q') - (a_2+2a_3)(\hat{m}+m_s) \\ &\quad +\frac{1}{2}(d_3+2d_5)q\cdot q' + \frac{1}{2}(d_4+2d_6)v\cdot qv\cdot q'. \qquad (7) \end{aligned}$$

Note that the leading order contribution – the first term in each amplitude – comes from the nucleon coupling to the vector current and is odd under crossing ($q\leftrightarrow -q'$). As it stands, it is attractive for K^-p at threshold. If this were the main story, it would be in disagreement with the repulsion observed in nature[19]. The cause for this is well-known [18]: The presence of $\Lambda(1405)$ slightly below the K^-p threshold results in the repulsion for the s-wave K^-p scattering overcoming the vector attraction. The effect of $\Lambda(1405)$ must therefore be taken into account.

Now how do we incorporate $\Lambda(1405)$ in chiral perturbation theory? As discussed in [6], since it is a bound state of the baryon-kaon complex[18][#1], it should be introduced as an independent heavy baryon field much like the resonance Δ (more generally the decuplet T^μ_ν). In particular it cannot be generated as a loop correction since the latter would involve "reducible graphs" involving infrared singularities, rendering the bound state inaccessible to chiral perturbation theory. Denoting the $\Lambda(1405)$ by Λ_R, we can write the leading-order Lagrangian density as

$$\mathcal{L}_{\Lambda_R} = \bar{\Lambda}_R(iv\cdot\partial - m_{\Lambda_R} + m_B)\Lambda_R + \left(\sqrt{2}\bar{g}_{\Lambda_R}\,\mathrm{Tr}\,(\bar{\Lambda}_R v\cdot A B_v) + \mathrm{h.\,c.}\right). \tag{8}$$

At tree order, the $\Lambda(1405)$ contributes only to the s-wave $K^\pm p$ scattering:

$$f^2\,T^{K^\pm p}_{v,\Lambda_R} = -\frac{\bar{g}^2_{\Lambda_R} v\cdot q' v\cdot q}{\frac{1}{2}[v\cdot(q-q')\mp v\cdot(q+q')] + m_B - m_{\Lambda_R}}. \tag{9}$$

Putting eqs.(6),(7) and (9) together, we obtain the complete $\mathcal{O}(Q^2)$ (*i.e.*, tree-order) s-wave $K^\pm p$ scattering lengths (see[20])

$$\begin{aligned}
a_0^{K^\pm p} &= \frac{m_N}{4\pi f^2(m_N+M_K)}[\mp M_K - \frac{\bar{g}^2_{\Lambda_R} M_K^2}{m_B \mp M_K - m_{\Lambda_R}} + (\bar{d}_s+\bar{d}_v)M_K^2] \\
a_0^{K^\pm n} &= \frac{m_N}{4\pi f^2(m_N+M_K)}[\mp \frac{M_K}{2} + (\bar{d}_s-\bar{d}_v)M_K^2],
\end{aligned} \tag{10}$$

where we have decomposed the contribution of order Q^2 – which is crossing-even – into a t-channel isoscalar ($\bar{d}_s$) piece and an isovector ($\bar{d}_v$) piece:

$$\begin{aligned}
\bar{d}_s &= -\frac{1}{2B_0}(a_1+2a_2+4a_3) + \frac{1}{4}(d_1+d_2+d_7+d_8) + \frac{1}{2}(d_3+d_4) + d_5 + d_6, \\
\bar{d}_v &= -\frac{1}{2B_0}a_1 + \frac{1}{4}(d_1+d_2+d_7+d_8),
\end{aligned} \tag{11}$$

with $B_0 = M_K^2/(\hat{m}+m_s)$.

The empirical scattering lengths[19, 22] are

$$\begin{aligned}
a_0^{K^+p} &= -0.31\,\mathrm{fm}, \qquad a_0^{K^-p} = -0.67 + i\,0.63\,\mathrm{fm} \\
a_0^{K^+n} &= -0.20\,\mathrm{fm}, \qquad a_0^{K^-n} = +0.37 + i\,0.57\,\mathrm{fm}.
\end{aligned} \tag{12}$$

[#1] It is a bound state of an $SU(2)$ soliton and an s-wave kaon in the Callan-Klebanov model [21].

The data for $K^\pm n$ are not well determined, so we cannot pin down the parameters in a quantitative way. Nonetheless, we can use these data to constrain the low-energy constants in our effective chiral Lagrangian. To tree order, the amplitudes are real, the imaginary parts of the amplitudes appearing at $\mathcal{O}(Q^3)$ involving loop graphs. In eq.(10), f is the meson decay constant in chiral limit and the difference between f_π and f_K is of order Q^4. Therefore we are allowed to simply take $f \approx f_\pi \approx 93$ MeV, the physical value. Now requiring consistency with the data at tree level leads to

$$\begin{aligned} (\bar{d}_s - \bar{d}_v)_{emp} &\approx (0.05 \sim 0.06)\,\mathrm{fm}, \\ (\bar{d}_s + \bar{d}_v)_{emp} &\approx 0.13\,\mathrm{fm}, \qquad \bar{g}^2_{\Lambda_R} = 0.15 \end{aligned} \tag{13}$$

with $m_{\Lambda_R} = 1.405\mathrm{GeV}$[#2]. Let us call these "tree-order empirical." Although it is difficult to make a precise statement due to the uncertainty in the data and the $\Lambda(1405)$ parameters, the above values provide a persuasive indication that *the net contribution of order Q^2 or higher for $K^\pm p$ ($K^\pm n$) is attractive and amounts to $\approx 33\,\%$ ($26 \sim 31\,\%$) of the strength given by the leading-order vector coupling.* This feature will be reconfirmed later at one-loop order.

Before going to the next order that involves loops, we discuss briefly what one can learn from the "tree-order empirical" eq.(13). The structure of the terms involved here is simple enough to render a relatively unambiguous interpretation. For this, first we note that the constants $\bar{d}_{s,v}$ consist of the KN sigma term $\Sigma_{KN} = -\frac{1}{2}(\hat{m} + m_s)(a_1 + 2a_2 + 4a_3)$ involving the quark mass matrix and the d_i terms containing two time derivatives. As argued in [6], the latter should be given by the leading $1/m_B$ correction in the heavy-fermion formalism with no renormalization by loop graphs. This can be readily seen from the chiral counting rule. The leading $1/m_B$ correction can be computed as explained in [6] from kaon-nucleon Born diagrams of relativistic chiral Lagrangians with the octet and decuplet intermediate states by taking the limit $m_B \to \infty$. Assuming flavor $SU(3)$ symmetry, one can calculate, in conjunction with the s-wave πN scattering lengths [#3]

$$\bar{d}_{s,\frac{1}{m}} = -\frac{1}{48}\left[(D+3F)^2 + 9(D-F)^2\right]\frac{1}{m_B} - \frac{1}{6}|\mathcal{C}|^2\frac{1}{m_B}$$

[#2] $\bar{g}^2_{\Lambda_R} = 0.15$ corresponds to $g^2_\Lambda/4\pi \approx 0.3$ in the conventional notation[19].

[#3] The decuplet contributions were omitted in [6] without justification. Here we rectify that omission and find that their contributions are essential. In calculating the decuplet contributions to the lowest order in $1/m_B$ from Feynman graphs in relativistic formulation, one encounters the usual off-shell non-uniqueness characterized by a factor Z [23] in the decuplet-nucleon-meson vertex when the decuplet is off-shell. Our result corresponds to taking $Z = -1/2$ consistent with s-wave πN scattering lengths at the same chiral order, $\mathcal{O}(Q^2)$.

$$\approx -0.55/m_B \approx -0.115\,\mathrm{fm}$$

$$\bar{d}_{v,\frac{1}{m}} = -\frac{1}{48}\left[(D+3F)^2-3(D-F)^2\right]\frac{1}{m_B}+\frac{1}{18}|\mathcal{C}|^2\frac{1}{m_B} \tag{14}$$

$$\approx 0.057/m_B \approx 0.012\,\mathrm{fm},$$

where the coupling $|C|^2(\approx 2.58)$ that appears in the decuplet contributions was fixed[#4] from $\Delta(1230) \to N\pi$ decay width. We have used $D = 0.81$ and $F = 0.44$ as determined at tree order from hyperon decays. The results eq.(14) together with the empirical values (13) imply that $\Sigma_{KN} \approx 2m_\pi$ which is consistent with a negligible strangeness content of the proton, $\langle P|\bar{s}s|P\rangle \approx 0$.

Since the next-to-leading order ($\mathcal{O}(Q^2)$) chiral corrections are not small, it is clearly important to go to the next order in the chiral expansion. At order Q^3, we have contributions from one-loop graphs given by Figure 1 and counter-term contributions from $\mathcal{L}^{(3)}$. The $\mathcal{O}(Q^3)$ corrections involving $\Lambda(1405)$ will be treated below. All the counter-term contributions to s-wave $K^\pm N$ scattering are crossing-odd and have incalculable low-energy constants. The crossing-even terms coming from the one-loop graphs involving $\mathcal{L}^{(1)}$ are finite whereas the crossing-odd terms from the same graphs are renormalized by the counter terms. Thus the s-wave $K^\pm N$ scattering lengths coming from the order Q^3 terms can be written as

$$\begin{aligned}
\delta_3 a_0^{K^\pm p} &= \frac{m_N}{4\pi f^2(m_N+M_K)}[(L_s+L_v)\pm(\bar{g}_s+\bar{g}_v)]M_K^3,\\
\delta_3 a_0^{K^\pm n} &= \frac{m_N}{4\pi f^2(m_N+M_K)}[(L_s-L_v)\pm(\bar{g}_s-\bar{g}_v)]M_K^3.
\end{aligned} \tag{15}$$

Here L_s (L_v) is the finite crossing-even t-channel isoscalar (isovector) one-loop contribution

$$\begin{aligned}
L_s M_K^3 &= \frac{1}{128\pi f^2}\Big\{+\frac{1}{3}(D-3F)^2(M_\pi^2+3M_\eta^2)M_\eta - 9M_K^2\sqrt{M_\eta^2-M_K^2}\Big\}\\
L_v M_K^3 &= \frac{1}{128\pi f^2}\Big\{-\frac{1}{3}(D+F)(D-3F)(M_\pi^2+3M_\eta^2)(M_\pi+M_\eta),\\
&\quad -\frac{1}{6}(D+F)(D-3F)(M_\pi^2+3M_\eta^2)(M_\pi^2+M_\eta^2)\int_0^1\frac{1}{\sqrt{(1-x)M_\pi^2+xM_\eta^2}}\\
&\quad -3M_K^2\sqrt{M_\eta^2-M_K^2}\Big\}.
\end{aligned} \tag{16}$$

One can show that the baryon decuplets do not contribute to the s-wave crossing-even amplitudes.[#5]

[#4] The decuplet-octet mass difference δ_T figures only in the denominator $1/(m_B+\delta_T)$, so it contributes at $\mathcal{O}(1/m_B^2)$ which we ignore here.

[#5] The diagrams (d), (e) and (f) in Figure 1 do contribute to crossing-even amplitudes. However the contributions

If we use $D = 0.81$ and $F = 0.44$, we obtain

$$\begin{aligned} L_s M_K &\approx -0.109\,\mathrm{fm}, \\ L_v M_K &\approx +0.021\,\mathrm{fm}. \end{aligned} \tag{17}$$

The quantity $\bar{g}_s$ ($\bar{g}_v$) in eq.(15) is the crossing-odd t-channel isoscalar (isovector) contribution from one-loop plus counter terms which after the standard dimensional regularization, takes the form

$$\bar{g} = \sum_i \gamma_i z_i^r(\mu) + \sum_i \left(\alpha_i \ln\left(\frac{m_i}{\mu}\right) + f_i(m_i) \right) \tag{18}$$

where the subscript i stands for π, K and η, z^r are linear combinations of renormalized coefficients of $\mathcal{L}^{(3)}$ defined at a scale μ, $f_i(m_i)$ are calculable μ-independent functions of m_i and γ_i and α_i are known constants. If there are enough experimental data, one may first fix the scale μ and then determine the unknown constants z_i. This would allow the same Lagrangian with the parameters so determined to make predictions for other processes, the power of effective field theories. For our purpose, this is neither feasible nor necessary. In fact, to $\mathcal{O}(Q^3)$ in the chiral counting, the quantity $\bar{g}$ of eq.(18) is μ-independent (or renormalization-group invariant). Therefore we will just fix $\bar{g}$ directly by experiments. Of course given $\bar{g}$, we can always re-express the first term of eq.(18) in terms of quantities fixed at some given μ as would be needed for comparisons with other processes.

Finally an equally important contribution that need to be considered at $\mathcal{O}(Q^3)$ is the loop effect on the property of the $\Lambda(1405)$. Because of the open channel $\Lambda(1405) \to \Sigma\pi$, the one-loop self-energy of the $\Lambda(1405)$ becomes complex, giving an imaginary part to the $\Lambda(1405)$ mass. If one assumes flavor $SU(3)$ symmetry which is valid at tree level (*i.e.*, to $\mathcal{O}(Q^2)$), then the $\Lambda(1405)\Sigma\pi$ coupling is the same as the $\Lambda(1405)pK$ coupling, so the width corresponding to the decay $\Lambda(1405) \to \Sigma\pi$ must be determined by the value $\bar{g}^2_{\Lambda_R} \approx 0.15$ obtained above. The corresponding width comes out to be

$$\Gamma_{\Lambda_R} \approx 50 \ \ \mathrm{MeV} \tag{19}$$

which is in agreement with the empirical width of $\Lambda(1405)$. This together with perturbative unitarity suggests that the amplitude (9) with m_{Λ_R} replaced by a complex mass could be used to take

of (d) and (f) cancel each other between the decuplet- and octet-K, $\pi^\pm$ intermediate states as shown in [24]. The non-vanishing contributions of (d) and (f) come only from the octet-π^0,η intermediate states. As for the graph (e), only the octet baryons contribute.

into account the $\mathcal{O}(Q^3)$ effect. However if we take the complex mass for m_{Λ_R}, the real part of eq.(9) becomes smaller. Hence, in order to compensate for the reduced amplitude, we found that it was necessary to take a larger effective coupling, $\bar{g}^2_{\Lambda_R} \approx 0.25$ at $\mathcal{O}(Q^3)$. This difference may perhaps be justified by the fact that $SU(3)$ breaking which enters at one-loop order could induce the coupling constants to differ by as much as 30% [25].

The results obtained by constraining $\bar{d}_s$, $\bar{d}_v$, $\bar{g}_s$ and $\bar{g}_v$ to the empirical data eq. (12) are

$$\begin{aligned} \bar{d}_s &\approx 0.201\,\mathrm{fm}, \qquad \bar{d}_v \approx 0.013\,\mathrm{fm}, \\ \bar{g}_s M_K &\approx 0.008\,\mathrm{fm}, \qquad \bar{g}_v M_K \approx 0.002\,\mathrm{fm}. \end{aligned} \tag{20}$$

If one takes eq.(14) for the $1/m_B$ corrections assuming that higher-order $1/m_B$ corrections do not modify the fit importantly, we can extract the parameter $\sigma_{KN} \equiv -(1/2)(\hat{m}+m_s)(a_1+2a_2+4a_3)$ needed to fit the scattering lengths (more precisely, $\bar{d}_s \approx 0.201$ fm)

$$\sigma_{KN} \approx 2.83 m_\pi. \tag{21}$$

This is not *by itself* the KN sigma term since loops renormalize it but this enhanced value may play a role in kaon condensation phenomena. Unfortunately complications due to the increased number of terms and off-shell ambiguities that are introduced at loop orders do not permit as simple an analysis as the one made above at tree level. Further work is required to pin down, for instance, the roles of the $1/m_B$ corrections and the KN sigma term in supplying information on the strangeness content of the proton. These complications do not, however, obcure the main thrust of our paper, which is that the attraction found at $\mathcal{O}(Q^2)$ in the KN interaction remains unaffected by the loop graphs and that $\langle P|\bar{s}s|P\rangle \approx 0$ is consistent with the s-wave scattering data.

The scattering amplitudes in each chiral order are given in Table 1. One sees that while the order Q and order Q^2 terms are comparable, the contribution of order Q^3 is fairly suppressed compared with them. As a whole, the subleading chiral corrections are verified to be consistent with the "naturalness" condition as required of predictive effective field theories. Using other sets of values of f, D and F does not change L_s and L_v significantly, hence leaving unaffected our main conclusion.

We now turn to off-shell s-wave K^- forward scattering off static nucleons. The kinematics involved are $t=0$, $q^2 = q'^2 = \omega^2$, $s = (m_N+\omega)^2$ with an arbitrary (off-shell) ω. In terms of the

$\bar{g}^2_{\Lambda_R} = 0.25$	$\mathcal{O}(Q)$	$\mathcal{O}(Q^2)$	$\mathcal{O}(Q^3)$	$\Lambda(1405)$
$a^{K^+p}(fm)$	-0.588	0.316	-0.114	0.076
$a^{K^-p}(fm)$	0.588	0.316	-0.143	-1.431
$a^{K^+n}(fm)$	-0.294	0.277	-0.183	0.000
$a^{K^-n}(fm)$	0.294	0.277	-0.201	0.000

Table 1: Scattering lengths from three leading order contributions.
Also shown is the contribution from $\Lambda(1405)$.

low-energy parameters fixed by the on-shell constraints, the off-shell K^-N scattering amplitude[#6] as a function of ω comes out to be

$$\begin{aligned}
a^{K^-p} &= \frac{1}{4\pi(1+\omega/m_N)}\Bigg\{ T_v^{K^-p}(\omega = M_K) - \frac{\omega^2}{f^2}\left(\frac{\bar{g}^2_{\Lambda_R}}{\omega + m_B - m_{\Lambda_R}}\right) \\
&\quad + \frac{1}{f^2}(\omega - M_K) + \frac{1}{f^2}(\omega^2 - M_K^2)\left(\bar{d}_s - \frac{\sigma_{KN}}{M_K^2} + \bar{d}_v + \frac{(\hat{m}+m_s)a_1}{2M_K^2}\right) \\
&\quad + \frac{1}{f^2}(L_p^+(\omega) - L_p^+(M_K)) - \frac{1}{f^2}(L_p^-(\omega) - L_p^-(M_K))\Bigg\}, \qquad (22)\\
a^{K^-n} &= \frac{1}{4\pi(1+\omega/m_N)}\Bigg\{ T_v^{K^-n}(\omega = M_K) \\
&\quad \frac{1}{2f^2}(\omega - M_K) + \frac{1}{f^2}(\omega^2 - M_K^2)\left(\bar{d}_s - \frac{\sigma_{KN}}{M_K^2} - \bar{d}_v - \frac{(\hat{m}+m_s)a_1}{2M_K^2}\right) \\
&\quad + \frac{1}{f^2}(L_n^+(\omega) - L_n^+(M_K)) - \frac{1}{f^2}(L_n^-(\omega) - L_n^-(M_K))\Bigg\}. \qquad (23)
\end{aligned}$$

[#6] The off-shell amplitude calculated here does not satisfy Adler's soft-meson conditions that follow from the usual PCAC assumption that the pseudoscalar meson field interpolate as the divergence of the axial current. This is because the meson fields of the chiral Lagrangian do not interpolate in the same way in the presence of explicit chiral symmetry breaking. However one can always redefine the meson fields consistently with chiral symmetry without changing the S-matrix so as to recover Adler's conditions. (Specifically this can be assured by imposing external gauge invariance in (pseudo-) scalar channel.) To the same chiral order, therefore, the two different (and all other equivalent) off-shell amplitudes have not only the same on-shell limit but perhaps also the same effective action at *the extremum point*. We do not have a rigorous proof of this statement but we believe it to be reasonable to assume that all physical quantities (including equation of state) computed from such an effective action would not depend upon how the meson field extrapolates off-shell, which is of course at variance with the arguments made by the authors in [8, 9]. This point was stressed to us by Aneesh Manohar.

Here[#7]

$$
\begin{aligned}
L_p^+(\omega) &= \frac{\omega^2}{64\pi f^2}\Big\{\Big[2(D-F)^2+\frac{1}{3}(D+3F)^2\Big]M_K+\frac{3}{2}(D+F)^2M_\pi \\
&\qquad +\frac{1}{2}(D-3F)^2M_\eta-\frac{1}{3}(D+F)(D-3F)(M_\pi+M_\eta) \\
&\qquad -\frac{1}{6}(D+F)(D-3F)(M_\pi^2+M_\eta^2)\int_0^1 dx\frac{1}{\sqrt{(1-x)M_\pi^2+xM_\eta^2}}\Big\} \\
&\quad +\frac{\omega^2}{8f^2}\left(4\Sigma_K^{(+)}(-\omega)+5\Sigma_K^{(+)}(\omega)+2\Sigma_\pi^{(+)}(\omega)+3\Sigma_\eta^{(+)}(\omega)\right), \\
L_p^-(\omega) &= \alpha_p M_K^2\omega+\beta_p\omega^3+\frac{1}{4f^2}\omega^2\left\{-\frac{1}{2}\Sigma_K^{(-)}(\omega)-\Sigma_\pi^{(-)}(\omega)-\frac{3}{2}\Sigma_\eta^{(-)}(\omega)\right\}, \\
L_p^-(M_K) &= (\bar{g}_s+\bar{g}_v)M_K^3, \\
L_n^+(\omega) &= \frac{1}{64\pi f^2}\omega^2\Big\{\Big[\frac{5}{2}(D-F)^2+\frac{1}{6}(D+3F)^2\Big]M_K+\frac{3}{2}(D+F)^2M_\pi \\
&\qquad +\frac{1}{2}(D-3F)^2M_\eta+\frac{1}{3}(D+F)(D-3F)(M_\pi+M_\eta) \\
&\qquad +\frac{1}{6}(D+F)(D-3F)(M_\pi^2+M_\eta^2)\int_0^1 dx\frac{1}{\sqrt{(1-x)M_\pi^2+xM_\eta^2}}\Big\} \\
&\quad +\frac{\omega^2}{8f^2}\cdot\left(2\Sigma_K^{(+)}(-\omega)+\Sigma_K^{(+)}(\omega)+\frac{5}{2}\Sigma_\pi^{(+)}(\omega)+\frac{3}{2}\Sigma_\eta^{(+)}(\omega)\right), \\
L_n^-(\omega) &= \alpha_n M_K^2\omega+\beta_n\omega^3+\frac{1}{4f^2}\omega^2\left\{\frac{1}{2}\Sigma_K^{(-)}(\omega)-\frac{5}{4}\Sigma_\pi^{(-)}(\omega)-\frac{3}{4}\Sigma_\eta^{(-)}(\omega)\right\}, \\
L_n^-(M_K) &= (\bar{g}_s-\bar{g}_v)M_K^3,
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
\Sigma_i^{(+)}(\omega) &= -\frac{1}{4\pi}\sqrt{M_i^2-\omega^2}\times\theta(M_i-|\omega|)+\frac{i}{2\pi}\sqrt{\omega^2-M_i^2}\times\theta(\omega-M_i), \\
\Sigma_i^{(-)}(\omega) &= -\frac{1}{4\pi^2}\sqrt{\omega^2-M_i^2}\ln\left|\frac{\omega+\sqrt{\omega^2-M_i^2}}{\omega-\sqrt{\omega^2-M_i^2}}\right|\times\theta(|\omega|-M_i) \\
&\quad -\frac{1}{2\pi^2}\sqrt{M_i^2-\omega^2}\sin^{-1}\frac{\omega}{M_i}\times\theta(M_i-|\omega|).
\end{aligned}
\tag{25}
$$

The results for K^-p and K^-n scattering are summarized in Figure 2 for the range of $\sqrt{s}$ from 1.3 GeV to 1.5 GeV with $\bar{g}_{\Lambda_R}^2=0.25$ and $\Gamma_{\Lambda_R}=50$ MeV. Those for K^-n scattering are independent of

[#7] The functions $L_{p,n}^-(\omega)$ contain four parameters $\alpha_{p,n}$ and $\beta_{p,n}$. Owing to the constraints at $\omega=M_K$, $L_{p,n}^-(M_K)$, they reduce to two. These two cannot be fixed by on-shell data. However since the off-shell amplitudes are rather insensitive to the precise values of these constants, we will somewhat arbitrarily set $\alpha_{p,n}\approx\beta_{p,n}$ in calculating Figure 2.

the $\Lambda(1405)$ and vary smoothly over the range involved. Our predicted K^-p amplitude is found to be in fairly good agreement with the recent fit by Steiner[17]. The striking feature of the real part of the K^-p amplitude, repulsive above and attractive below $m_\Lambda(1405 MeV)$ as observed here, and the ω-independent attraction of the K^-n amplitude are quite possibly relevant to kaonic atoms [26] and to kaon condensation in "nuclear star" matter. In comparison with Steiner's results, the imaginary part of the K^-p amplitude predicted here is a bit too big. This may be due to our approximation of putting the experimental decay width of $\Lambda(1405)$ for the imaginary part of its mass.

The calculation of the effective action at loop orders needed for kaonic atom and kaon condensation phenomena will be reported elsewhere.

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Figure Captions

- **Figure 1:** One-loop Feynman diagrams contributing to $K^{\pm}N$ scattering: The solid line represents baryons (nucleon for the external and octet and decuplet baryons for the internal line) and the broken line pseudo-Goldstone bosons ($K^{\pm}$ for the external and K, π and η for the internal line). There are in total thirteen diagrams at one loop, but for s-wave KN scattering, for reasons described in the text, we are left with only six topologically distinct one-loop diagrams.

- **Figure 2:** $K^{-}N$ amplitudes as function of $\sqrt{s}$: These figures correspond to eqs.(23) and (24) with $\bar{g}^2_{\Lambda_R} = 0.25$, $\Gamma_{\Lambda_R} = 50$ MeV and $\alpha_{p,n} = \beta_{p,n}$, fixed in the way described in the text. The first kink corresponds to the KN threshold and the second around 1.5 GeV to $\sqrt{s} = m_N + M_\eta$ for $M_\eta \approx 547$ MeV.